

**University of Halmstad**  
**Master's Programme for Computer Science**  
**and Electrical Engineering**

Laboratory for  
**IMAGE ANALYSIS 1**

**Project**

Topics of the project work:

**- The Structure Tensor in 2D**

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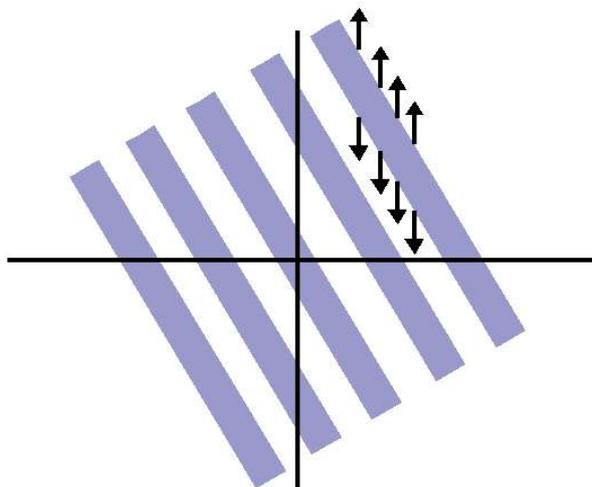
# 1 Introduction

This project is part of the lecture "Image Analysis I". It deals with the topic of the structure tensor in 2D.

The theory beyond the term linear symmetry has to be mentioned in order to give a better understanding of the whole topic. In general, **ideal** linear symmetry means that

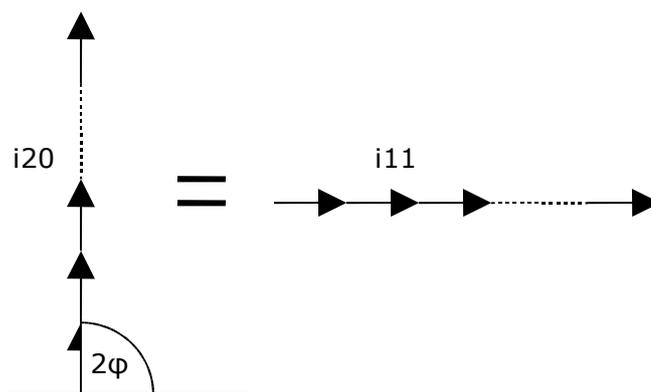
$$\text{abs}(i_{20}) = i_{11}.$$

This rather unclear term can be best described by a simple depiction of a line pattern, where ideal linear symmetry appears:



**Figure 4: Line Pattern With Gradients**

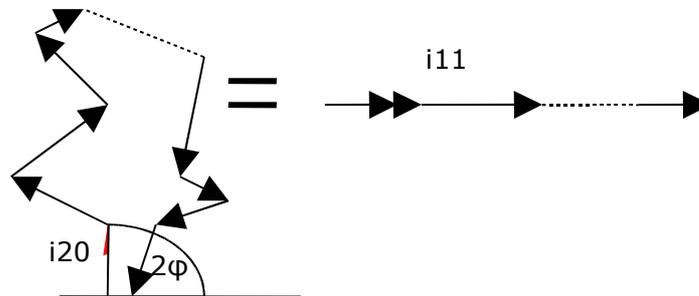
The summation of the lengths of the gradients has to be the same as the gradients strung together what is the case here:



**Figure 5: Gradients Strung Together and Their Sum (IDEAL Linear Symmetry)**

As it can be seen from the figure above, the angle of  $i_{20}$  is  $2\phi$ . The magnitude is the same as  $i_{11}$ . This automatically means that this must be the case of ideal linear symmetry.

The next possibility is **no** linear symmetry. This happens, if all gradients have different directions and a very small magnitude, as shown in the following figure:



**Figure 6: Gradients Strung Together and Their Sum (NO Linear Symmetry)**

No linear symmetry can be found in the case when talking e.g. about noise, where the gradients are randomly ordered in the "image" and their magnitudes are very small. In this case, the absolute value of  $i_{20}$  is much smaller than  $i_{11}$ .

The third possibility regarding linear symmetry is a **constant image**. In this case, the absolute value of  $i_{20}$  as well as  $i_{11}$  are both 0. This results out of the fact that there are no gradients in a constant picture, meaning that there are no frequencies. This part will be treated later on in more detail.

## 2 Outcomes

In order to employ these facts mentioned above, we made several pictures and calculated their power spectrum.

The MATLAB commands for this operation looked like as follows:

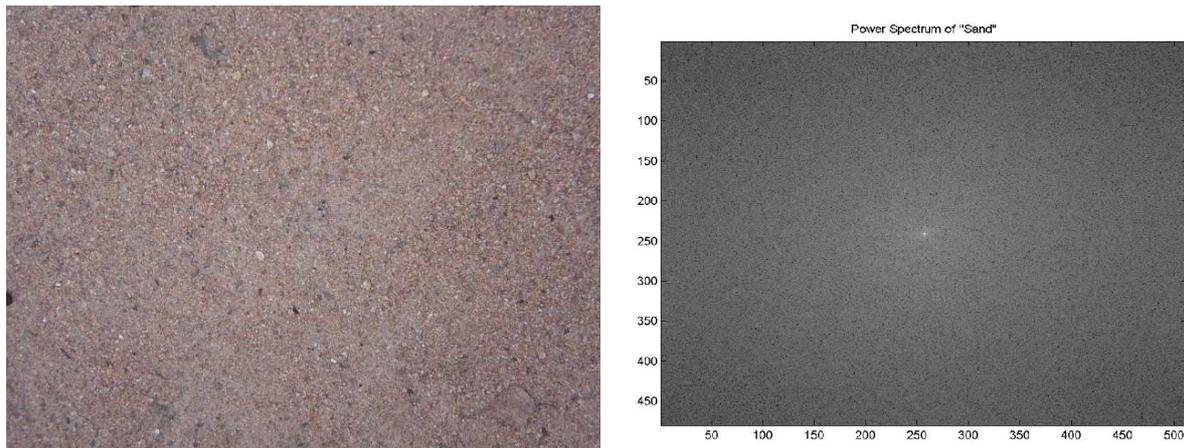
```
clear all;
close all;

%load image and find orientations
f=double(imread('jalousie.jpg'));
f=f(1:480,1:480);
LS=linsymexer(f,[1.5,2]);
figure(1);
imagesc(f); truesize; colormap(gray); title('Jalousie');

%apply Hamming window
hx=hamming(480);
h=hx*hx';
hf=h.*f;
figure(2);
imagesc(hf); truesize; colormap(gray); title('Jalousie');

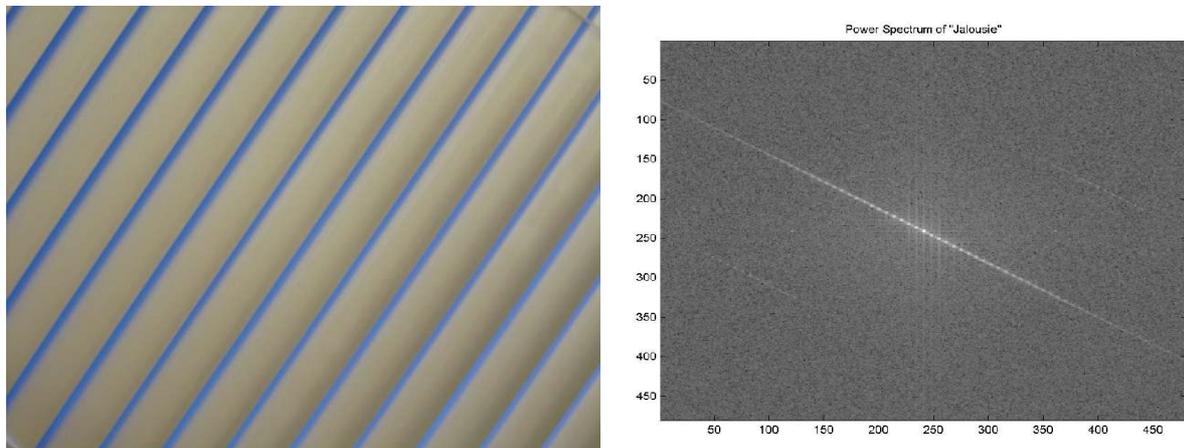
%produce power spectrum
c=fft2(hf);
p=abs(c).^2;
figure(3);
imagesc(fftshift(log10(p))); colormap(gray); title('Power Spectrum of
"Jalousie");
```

The following figure depicts a sand image and its power spectrum. It is obvious that no linear symmetry exists in the picture. The image size is 480x512.



**Figure1: Picture of Sand and its Power Spectrum**

The following figure shows an extract of blinds. From the power spectrum, it can be seen that in the picture there is linear symmetry:

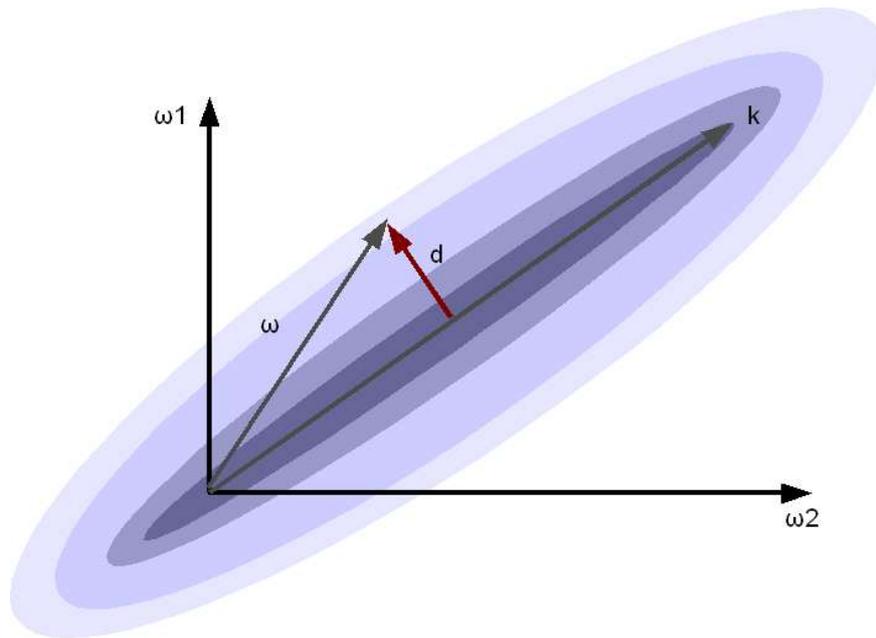


**Figure2: Picture of Blinds and its Power Spectrum**

The diagonal line in the power spectrum represents the direction vector of the orientation in the picture. As it can be seen in the spectrum, there are some vertical lines, which could be minimized by laying a 2D-hamming window over the whole picture in order to compensate fluctuations of the same colour in the original image. This means that e.g. the gaps between the blinds do not always have exactly the same colour. This problem could have been solved by converting the image to a black/white picture what is done by setting a threshold within the gray values, with which each pixel can be assigned to a certain group, in our case the gaps or the blinds themselves. This method was not employed in order to get the real power spectrum of the picture and not to distort it too much because of such operations.

In the next part of the assignment, we should create a figure that depicts an axis fitting to a plate with non-uniform mass distribution represented by  $|F(\omega_x, \omega_y)|^2$ . The vector  $k$  illustrates the fitted axis, the  $d(\omega, k)$  illustrates the distance between the mass density  $|F(\omega_x, \omega_y)|^2$  and the axis  $\omega$ .

Our depiction of this task is shown in the following figure:



**Figure3: Error Function with Direction Vector  $k$**

In the figure above,  $\omega$  is a vector (point) and  $d$  is the normal distance to the direction vector  $k$ . The error function looks like this:

$$|e(k) = \int d^2(\omega, k) \cdot |F(\omega)|^2 d\omega$$

The vector  $k$  is obtained by minimizing this function. Therefore, the method of least squared errors is employed. This line described above could also be seen as a plate regarding it in three dimensions. The procedure of the least error squares for computing it remains the same.